SFC ディスカッションペーパー
SFC-DP 2013-001

A Micro-based Econometric Model Determining Price/Wage Rate/Factor Demands of Interindustry Economies

小坂弘行

総合政策学部名誉教授

hkosaka@sfc.keio.ac.jp

2013 年 9 月

A Micro-based Econometric Model Determining Price/Wage Rate/Factor Demands of Interindustry Economies

Hiroyuki Kosaka
Faculty of Policy Management, Keio University
Endo 5322,Fujisawa,Kanagawa,252-8520,Japan
E-mail: hkosaka@sfc.keio.ac.jp

Acknowledgments
I am indebted to for commentators including Prof.P.Nijkamp(Free University, Netherlands) of my presentations at related academic meetings; yet, remaining errors are all mine.

1. Introduction

After the end of the Cold War, individual economy goes into the era of mega competition; most industries including even those of services are subject to international competition as are observed not only in traded goods, but also in non-traded goods. Input output table depicts detailed map of complexity of nexus among industries. Composition of input output table shows national economy in aggregation by way of figuring out micro-flow among industries; micro-economies fuses into macro-economy getting entangled with each other.

Modeling multi-sector system, which has been pioneered by W.Leontief, has two kinds of modeling frameworks; a) provided both demand and supply depend on price, adjustment of gap of demand/supply is mainly done by price in static or dynamic way, i.e. the system is called price adjustment scheme; b) but, when market price has insufficient power of adjusting the gap, mainly supply meets demand, i.e. the system called quantitative adjustment scheme. When supply is of price-rigidity, demand can be controlled to meet supply by price; the case is also in price adjustment. In an aspect of price determination, while market determines price in former scheme, producer determines price in the latter scheme. The two schemes are extreme; real industrial economies are located between the two extremes.

We briefly review preceding studies on interindustry economy on a viewpoint of adjustment schemes.

a) Price Adjustment Scheme

As an example, the first contribution by econometrics using multi-period data would be made by M.Saito(1971); he designed multi-sector system on an extension of cobweb in micro-economics; intermediate demands follows Leontief's fixed input coefficient, other factor demands from profit maximization under perfect competition, and price adjusts demand/supply in dynamic way, resulting in zero profit. And just after M.Saito(1971), J.B.Shoven and J.Whalley(1972,1973) have made price iteration calculation to meet demand/supply following their teacher H.E.Scarf modified algorithm ; group of J.B.Shoven and J.Whalley is school of applied general equilibrium(AGE). And, S.Robinson constitutes another school, called CGE group; see e.g. H.Lofgren,R.L.Harris and S.Robinson(2002).

b) Quantitative Adjustment Scheme

Typical one of this category is by W.Leontief(1951); his model of demand/supply, both being independent of price, depicts that supply meets demand perfectly, and that price is determined by cost accounting. Works of the first generation econometric models followed W.Leontief; outstanding example of large scale model is the Brookings Model by J.S.Duesenberry,G.Fromm,L.R.Klein and E.Kuh(1965); the model is fructified to Wharton Long-Term Model by Wharton Econometric Forecasting Associates(1982). Even tough these econometric models are out of consideration in academic field, model analysis for simulation and prediction in practical application is still alive; because of lack of micro-foundation[1] in model building, micro-building effort is needed. Incidentally, so-called "IO technique" in the absence of price is in this category. Main characteristics of the scheme is that price arises from

[1] Micro-foundation of modeling requires to expose the followings: a) decision makers; b) objective functions to be optimized by decision makers, e.g. household utility, firm's profit, and so on; c) equations derived by optimization of decision makers. Traditional econometric modeling lacks the step of b), then it expose equations a prior.

cost-related accounting, and it adjusts demand/supply in part. Micro-founded modeling requires to assign profit maximization to derive price, not by market clearance. A typical example of AGE model by C.L.Ballard,D.Fullerton,J.B.Shoven and J.Whalley(1985) is demand oriented, and has price by cost-related accounting.

Now at present, main stream of multi-sector modeling might be CGE. Yet, model building seems to be developing longing for two directions; a) econometric estimation employing multi-period data; b) employing imperfect competition apart from Walrasian perfect competition. The reason of employing econometric modeling would lie in models having plural parameters for describing behavioral equations in stable way, which equations might be substantially dynamic, but in no static nature; so that, total story of economic phenomena completes in no single period.

c) Econometric Estimation and Imperfect Competition

Recent trend has been directed to econometric modeling; L.Johansen(1960), origin of current CGE, stated the following in the third paragraph of Chapter four, i.e.,The Input-Output Table.

For a system as large as ours, it would be an overwhelming task to estimate all coefficients by sophisticated statistical methods based on time series. Furthermore, time series are not in fact available, and, if they were, so many years would need to be used that the structure would have significantly changed over the period. We are therefore forced to rely on rougher methods of the type commonly used in connection with input-output analysis. In fact, the model has been intentionally constructed in such a way that it is possible to implement it numerically mainly with data from only a single year.

Reflecting L.Johansen(1960), econometric estimation of parameters of CGE is becoming active; e.g. D.W.Jorgenson(1984), D.W.Jorgenson and K.Y.Yun(1990)， D.W.Jorgenson and D.T. Slesnick(1997)， McKendrick(1998). And Arndt,C.,S.Robinson and F.Tarp(2002) argued CGE model estimation by entropy maximization. D.W.Jorgenson(2012) has summarized the recent econometric estimation for CGE.[2] On the other hand, modeling of imperfect competition follows; an early example of AGE model treating international trade is found in R.Harris(1984); another early example of econometric model is K.Tsujimura and M.Kuroda(1974) in the presence of imperfect competition, but unfortunately unknown outside of Japan.

d) Introducing Cost Function

Since 2000, modeling of multi-sector system has new movement; assume cost function accommodating production function of scale economy, and derive factor demands by Shephard' Lemma by way of cost function; e.g. K.Kratena(2005) and W.E.Diewert and K.J.Fox(2008) incorporated cost function for modeling multi-sector system.[3] By doing this, price formation and also wage rate could be described by profit maximization; parameter estimation could be carried out by econometric way. Modeling of multi-sector system is directed to imperfect competition, and econometric estimation using multi-period data. This article is according the line. In line with this, H.Kosaka(2011) modeled multi-country/multi-sector system, and T.Shibata and H.Kosaka(2011) domestic multi-region/multi-sector system, both arguing price

---

[2] Consumer/producer/price setting behaviors are all approximated by transglog function.

[3] K.Kratena(2005) employed generalized Leontief cost function for deriving intermediate demands; but, price is cost-related accounting.

formation in virtue of conjectural variation.

Price movement would be dependent considerably on the past with no rapid move as are encountered in price-rigidity of macro-economics, and in numerical example in J.B.Shoven and J.Whalley(1973); such phenomena should be reflected in price determination formulation. The present paper attempts to elucidate dynamic price formation in connection with profit maximization.

Hence, this paper takes full information of long-term data into modeling in a unified platform; a) to build an econometric model on a base of micro-foundation; b) to make clear the competitiveness of the several markets for validity of the present model. For this purpose several industries of the Japanese economy, but not the whole industries, are taken into accounts. Factor demands and price/wage rate formation of multi-sector system are elucidated by the same principle of profit maximization; first we assume cost function, and estimate it numerically by estimating factor demand equations; in the second we estimate both marginal and average costs, and calculate scale economy; in the final step we estimate dynamic process of price movement by marginal cost in addition to wage rate. Additionally Rahner’s index for monopoly, TFP and rate of technical progress are also easily calculated. The modeling feature of this paper is describing all the relevant variables of the producer’s behavior in multi-sector system by profit maximization; undoubtedly cost function enables us to unveil the proper mechanism.

Section two explains model on producer’s behavior; then, section three data and empirical results; and finally we conclude.

2. Producer's Behaviour of Multi-sector System

2.1 Market of j-th Industry and Three Agents Model of the Firm

When one face with market, first one should distinguish the market is in equilibrium or in disequilibrium. Importance of disequilibrium is that disequilibrium but not equilibrium affects resource allocation; e.g. market of bank loan or Keynesian aggregated market. This paper assumes market is in equilibrium and imperfect. If disaggregated j-th market of national economy is in excess-demand($X_j^D > X_j^S$: X_j^D :demand, X_j^S :supply), price under full operation will adjust demand in part, and demand will be cut off; inversely, if market is in excess-supply($X_j^D < X_j^S$), producer will decrease operation to meet demand; in both cases, unplanned inventory works for absorbing the demand/supply gap. Above story is on production to stock in manufacturing industry. Ordered production in manufacturing industry always meets demand; therefore, production in process becomes inventory in process. Service industry differs from that of manufacture; it inevitably becomes ordered production because of no capability of inventory; e.g. banking service makes service in response to customers’ requests.

We focus on producer’s behavior; factor demands, price/wage formation are under consideration by the same principle. We assign cost function to determine factor demand via Shephard’s Lemma; and assign profit maximization to determine wage/price. It would be possible to assume monopolistic firms; but, this paper assume monopoly firm to avoid dispersion of firms within the market.

We post behaviors of producer; i.e. we interpret factor demands and price/wage rate in the cells of column vector of input output table under profit maximization. First, set long-term cost function having capital inside, then goes to factor demands and price/wage rate formation. Now it is essential to connect cost function with profit maximization; in this sense, we are in the same line of W.E.Diewert and K.J.Fox(2008), also early version W.E.Diewert and K.J.Fox(2004).[4] They take monopolistic firms; but, we take monopoly firm.

Consider j-th industry's profit, and develop Bertrand competition with price/wage rate being as instruments. Demand X_j^D and supply X_j^S are clearly distinguished. We have three agents inside of a firm; they cooperate each other leading to production behaviors. Upper agents (agent-P plus agent-W) plan to determine price/wage rate respectively under profit maximization; on the other hand, lower agent (agent-C) seeks to cost minimization, also consistent with profit maximization, under restriction of production function. First we explain cost minimizing behavior.

2.2 Determining Factor Demands by Agent-C

Given set of prices (plus wage rate) and order of production X_j^S, determine factor demands.

$$C_j(X_j^S, p, w_j, r, t) = \min_{\tilde{x}_j, L_j, K_j} \left(\sum_{i=1}^{N} p_i x_{ij} + w_j L_j + rK_j \right) \quad j = 1,.., N \qquad (2.1)$$

C_j: j-th industry cost function

x_{ij}: i-th input for j-th industry L_j:labor input K_j:capital input

$\tilde{x}_j = \{x_{ij}; i = 1,2,.., N\}$ p_i: price of j-th industry

Set of prices $p = \{p_i; i = 1,.., N\}$ w_j^k: wage rate of j-th industry r: capital cost

On the mathematical form of (2.1), many cost functions are posed. On account of no special kind of movement in cost function, L.R.Christensen,D.W.Jorgenson and L.J.Lau(1973) proposed translog cost function based on Taylor series expansion; nowadays the function has been widely used. However, cost function should have economic rationale; generalized Leontief cost function by M.Fuss(1977) would be remarked by that the cost function is generalized from Leontief constant input coefficient; the mathematical form is $C(y,p,t) = \sum_{i,j} h_{ij}(y,t)\sqrt{p_i}\sqrt{p_j}$ (p_i :i-th facor price, y :production, $h_{ij}(y,t)$:symmetric and concave). As M.Fuss(1977) did not specify $h_{ij}(y,t)$, many forms have been proposed later.

As we intend to investigate the Japanese interindustry economy for the past fourty years, we take long-term cost function in which capital adjustment are accomplished within the period; however, it would be possible to introduce capital adjustment like R.S.Pindyck and

---

4 W.E.Diewert and K.J.Fox(2008) model multi-sector system on the way of calculating technical progress.

J.J.Rotemberg(1983).[5]

Function $h_{ij}(y,t)$ must be specified for empirical study; we follow S.Nakamura(1990).

$$C(y,p,t)=\left[\sum_{i} b_{ii}p_i y^{b_{yi}}\exp(b_{ti}t)+\sum_{i\neq j} b_{ij}\sqrt{p_i p_j}\,y^{b_y}\exp(b_t t)\right] \tag{2.2}$$

$$h_{ii}(y,t)=b_{ii}y^{b_{yi}}\exp(b_{ti}t) \qquad i=1,2,..,m$$

$$h_{ij}(y,t)=b_{ij}y^{b_y}\exp(b_t t) \qquad i,j=1,2,..,m$$

p_i : price of i-th input y :production t :time trend

We focus on diagonal elements of the above quadratic form; turning to our cost function, the function of j-th industry is written down:

$$C_j(X_j^S,p,w_j,r,t)=\sum_{i=1}^{N} b_j^{ii}p_i(X_j^S)^{b_j^{Xi}}\exp(b_j^{ti}t)$$

$$+b_j^L w_j(X_j^S)^{b_j^{LX}}\exp(b_j^{Lt}t)+b_j^K r(X_j^S)^{b_j^{KX}}\exp(b_j^{Kt}t). \tag{2.3}$$

Furthermore, put expectation of current price p_j^e to the cost function which S.Shishido and O.Nakamura(1992) once has stressed for expressing innovation; factor inputs are reduced by process innovation which is caused by the reduction of the current price in price competitiveness with overseas, or potential competitive pressure even for monopoly firm .[6]

$$C_j(X_j^S,p,w_j,r,t)=\sum_{i=1}^{N} b_j^{ii}p_i(X_j^S)^{b_j^{Xi}}\exp(b_j^{ti}t)(p_j^e)^{b_j^{pi}}$$

$$+b_j^L w_j(X_j^S)^{b_j^{LX}}\exp(b_j^{Lt}t)(p_j^e)^{b_j^{Lp}}+b_j^K r(X_j^S)^{b_j^{KX}}\exp(b_j^{Kt}t)(p_j^e)^{b_j^{Kp}} \tag{2.4}$$

The use of Shephard's Lemma to (2.4) yields optimal intermediate demand $\partial C_j/\partial p_i$, optimal labor demand $\partial C_j/\partial w_j$ and optimal capital demand $\partial C_j/\partial r$. Note that the cost function can be known indirectly via estimated coefficients of factor demands; estimation is implemented on $X_j^S=X_j^D=X_j$.

[5] Cost function having all the optimal levels of factor demands are realized within a period is called long-term cost function; the equilibrium is in Full Static Equilibrium. Cost function having fixed inputs left inside as parameters like production is a short-term cost function, and is called restricted cost function; the equilibrium is in Partial Static Equilibrium. Cost function having fixed inputs partially realized is also a short-term; but, the equilibrium is in Partial Dynamic Equilibrium. R.S.Pindyck and J.J.Rotemberg(1983) is the third case.

[6] S.Shishido and O.Nakamura(1992) distinguished three kinds of technical progresses; the first is Hicks neutral, expressed by time; the second is that price change of particular industry affects factor demands of the other industries; both effects are expressed in Stone's RAS. The notable is the third; price competitiveness of mega competition era after 90s affects all the factor demands via reduction of current product price; the last effect can be called Shishido effect.

2.3 Dynamics of Price Formation by Agent-P

Cost function can be known once factor demands are estimated; then, the corresponding marginal cost also is known theoretically and numerically. Theoretical monopoly price is calculated from marginal cost via profit maximization if demand curve is given; but unfortunately, calculated monopoly price does not coincide with the actual price.[7] This implies actual price could not be reduced from calculated theoretical monopoly price via profit maximization; hense, to interpret the actual price, we need to introduce another mechanism to profit maximization. T.Shibata and H.Kosaka(2011) introduced conjectural variation by R.Frisch into profit maximization in the Japanese nine interregional input output model.[8]

However, this paper proposes a mechanism of dynamic price formation in the nexus of profit maximization in place of conjectural variation. Now, profit maximization is stated: $\max_{p_j} \pi_j^{(p)} = \max_{p_j} \{p_j X_j^D - C_j(X_j^S, p, w_j, r, t)\}$ in which X_j^D stands for "perceived demand function" in Negishi's sense (see T.Negishi(1961)); so that, Negishi's demand function is to be attached to profit maximization calculation.

Once A.W.Phillips(1954) proposed kinds of policy response functions in a macro-stabilization of multiplier-acceleration model; i.e. proportionate, derivative and integral policies; social welfare function, a pair of policy response function, has variations including quadratic loss.[9] Turning to our argument, the meaning of A.W.Phillips' is to introduce quadratic loss of price into profit; so that, maximize (unit) profit plus quadratic loss of price, then determine price in dynamism. Why does monopoly firm consider price change apart from profit itself ? Price which is derived by profit maximization may have fluctuation in violence[10]; restricted price, inversely, may damage optimality of profit maximization. Unfortunately, society could accept no drastic change of price. So that, monopoly firm, by reason of monopoly, ought to face severe eye of society, and avoids drastic change of price; this is considered social cost of monopoly firm. It should be noted, at the same time, that firm could control price imperfectly; it is affected by social, political, ecological, and even demographic factors besides economic one.

Now, consider price setting behavior unfavorable to drastic change of price; price change from the past is measured by $(p_j - p_{j,-1})$; then, the quadratic loss of price of difference is

---

[7] Monopoly price, which is calculated from coefficients of both demand curve of the current product and cost function, and the actual price are compared; as a result, arguments of demand side and supply side must be in isolation.

[8] Conjectural variation is usually introduced to price change of rival firm in response to that of the current firm; but, T.Shibata and H.Kosaka(2011) introduced conjectural variation into inventory change in response to price change within the monopoly firm.

[9] Among them, integral policy is close to ours; it has social welfare function in discrete case $f = w(Y - Y^*)^2 + (G - G_{-1})^2$ (Y :endogenous; G :instrument), and has corresponding policy response function $G - G_{-1} = -w_i(Y - Y^*)\partial Y / \partial G$.

[10] Long ago, G.C.Chow(1973) stressed the same issue in optimal control as "instrument instability."

expressed as a quadratic loss $\left(p_j - p_{j,-1} - \tilde{c}_j\right)^2 (\tilde{c}_j \neq 0)$ in general. Furthermore it would be plausible, in practical estimation, to relax strict difference to $(p_j - c_j^{p2} p_{j,-1} - \tilde{c}_j)^2 (0 < c_j^{p2} \leq 0)$. Then, profit maximization with quadratic loss of price is restated:

$$\max_{p_j} \tilde{\pi}_j^{(p)} = \max_{p_j} \left\{ -\frac{1}{2} c_j^{p1} (p_j - c_j^{p2} p_{j,-1} - \tilde{c}_j)^2 + \frac{1}{X_j^*} \left(p_j X_j^D - C_j\right) \right\}. \tag{2.5}$$

X_j^*: normal level of production with assuming $\partial X_j^* / \partial p_j = 0$

As profit is evaluated in nominal term, it is denominated by X_j^* to meet price variable. First order condition yields with assuming $\partial w_j / \partial p_j = 0$:

$$\frac{\partial \tilde{\pi}_j^{(p)}}{\partial p_j} = -c_j^{p1}\left(p_j - c_j^{p2} p_{j,-1} - \tilde{c}_j\right) + \frac{1}{X_j^*}\left(p_j \frac{\partial X_j^D}{\partial p_j} + X_j^D - MC_j \frac{\partial X_j^S}{\partial p_j} \right) = 0 \tag{2.6}$$

Rearranging term of price gives us dynamic equation of price determination:

$$p_j = \tilde{c}_j + c_j^{p2} p_{j,-1} + \frac{1}{c_j^{p1}} \times \left(\frac{1}{X_j^*} \right) \times \left(X_j^D + p_j \frac{\partial X_j^D}{\partial p_j} - MC_j \frac{\partial X_j^S}{\partial X_j^D} \frac{\partial X_j^D}{\partial p_j} \right) \tag{2.7}$$

On specifying X_j^D on demand side, it would be preferable to formulate demand allocating wealth over multiple goods; e.g. sophisticated demand model like A.S.Deaton and J.Muellbauer(1980) is applicable.[11] Yet, as is noted, T.Negishi's perceived demand function is supposed to have $\partial X_j^D / \partial p_j = -\beta_j \times \left(X_j^D / p_j\right)$ in elasticity form.[12] Next, unit increase of demand may increase supply in response to price; then, we suppose $\partial X_j^S / \partial X_j^D = \delta_j p_j (\delta_j > 0)$. Two relations are inserted into (2.7):

$$p_j = \tilde{c}_j + c_j^{p2} p_{j,-1} + \frac{1}{c_j^{p1}} \times \left(\frac{1}{X_j^*} \right) \times \left(X_j^D - \beta_j X_j^D + \beta_j \delta_j MC_j X_j^D \right) \tag{2.8}$$

In a final step, by putting $X_j^* = X_j^D$ [13], dynamic price equation for estimation is obtained.

$$p_j = \tilde{c}_j + \frac{1-\beta_j}{c_j^{p1}} + c_j^{p2} p_{j,-1} + \frac{\beta_j \delta_j}{c_j^{p1}} MC_j \tag{2.9}$$

---

[11] See, e.g. I.Mongelli, F.Neuwahl and J.M.Rueda-Cantuche(2010).

[12] As X_t^D is no actual demand, but is anticipated by monopoly firm, so that X_t^D should be $X_t^{D(S)}$; now, $X_t^{D(S)}$ is demand anticipated by supplier.

[13] It is possible to put $X_j^* = X_{j,-1}^D$, or to put $X_j^* = (X_j^D + X_{j,-1}^D)/2$.

As R.E.Hall(1988) has remarked, monopoly price may have distortion in spite of $p_j = MC_j$ in perfect competition. Anticipated price elasticity β_j by monopoly firm affects price determination in the two directions; the first term $(1-\beta_j)/c_j^{p1}$ in (2.9) is interpreted as pressure for reducing price that agent-P has to care; on the other hand, price change may urge supply change, which may urge additional cost $\beta_j \delta_j MC_j / c_j^{p1}$ leading price increase as a result of agent-C behavior.

Now, relaxed price difference is assumed to split into deterministic and stochastic parts:

$$\tilde{c}_j = \mu_j + \nu_j . \quad \nu_j \sim N(0, \sigma_j^2) \tag{2.10}$$

Equation (2.9) is , then, converted to:

$$p_j = \frac{\left(c_j^{p1}\mu_j + 1 - \beta_j\right)}{c_j^{p1}} + c_j^{p2} p_{j,-1} + \frac{\beta_j \delta_j}{c_j^{p1}} MC_j + \nu_j \tag{2.11}$$

Firm's price, affected by various factors other than economic one, become stochastic in part. Unknown parameters $(c_j^{p1}, c_j^{p2}, \mu_j, \beta_j, \delta_j)$ is undetermined even if we apply regression to (2.11); but, if two parameters are restricted, all parameters become known.

From (2.9) or (2.11), increase of marginal cost gives rise to increase of price; secondly increase of scale economy SE_j $(SE_j = AC_j / MC_j)$ decreases price; thirdly increase of Rahner's index of monopoly $LX_j = (p_j - MC_j)/p_j$ increases price.

2.4 Dynamic Process of Price Movement

All the terms of (2.4) are dependent on price. Marginal cost is easily obtained from cost function (2.4) in case of absence of Shishido effect.

$$MC_j(X_j^S, p, w_j, r, t) = b_j^{jj} b_j^{Xj} p_j (X_j^S)^{b_j^{Xj}-1} \exp(b_j^{tj} t) + \sum_{i \neq j}^{N} b_j^{ii} b_j^{Xi} p_i (X_j^S)^{b_j^{Xi}-1} \exp(b_j^{ti} t)$$
$$+ b_j^L b_j^{LX} w_j (X_j^S)^{b_j^{LX}-1} \exp(b_j^{Lt} t) + b_j^K b_j^{KX} r (X_j^S)^{b_j^{KX}-1} \exp(b_j^{Kt} t) \tag{2.12}$$

Uniting (2.12) with (2.11) gives us the following with $X_j^S = X_j$:

$$p_j = \frac{\left(c_j^{p1}\mu_j + 1 - \beta_j\right)}{c_j^{p1}} + c_j^{p2} p_{j,-1} + \frac{\beta_j \delta_j}{c_j^{p1}} \left(\frac{b_j^{jj} b_j^{Xj} \exp(b_j^{tj} t)}{X_j^{1-b_j^{Xj}}} \right) p_j$$

$$+\frac{\beta_j\delta_j}{c_j^{p1}}\left\{\sum_{i\neq j}^{N} b_j^{ii} b_j^{Xi}\left(\frac{\exp(b_j^{ti}t)}{X_j^{1-b_j^{Xi}}}\right)p_i + b_j^{L} b_j^{LX}\left(\frac{\exp(b_j^{Lt}t)}{X_j^{1-b_j^{LX}}}\right)w_j\right.$$

$$\left. + b_j^{K} b_j^{KX}\left(\frac{\exp(b_j^{Kt}t)}{X_j^{1-b_j^{KX}}}\right)r\right\} + v_j \qquad (2.13)$$

The third term of (2.13) comes from input of the current product; accordingly, disaggregation of industry will force to vanish. The fourth term comes from the other prices $p_i (i \neq j)$; the fifth from wage rate w_j of the current industry; the sixth is brought in from capital cost r. To sum up, the fourth, fifth and sixth terms are all brought in from the external; these terms become non-homogeneous part of auto-regressive process (2.13). Then, homogeneous part of (2.13) is:

$$\left\{1-\frac{\beta_j\delta_j}{c_j^{p1}}\left(\frac{b_j^{jj} b_j^{Xj}\exp(b_j^{tj}t)}{X_j^{1-b_j^{Xj}}}\right)\right\}p_j = \frac{\left(c_j^{p1}\mu_j + 1 - \beta_j\right)}{c_j^{p1}} + c_j^{p2} p_{j,-1} + v_j \qquad (2.14)$$

Note that coefficient of p_j is dependent on time; but, time dependent term of the coefficient will vanish. Convert variable $\tilde{p}_t = p_t - a_0/(1-a_1)$ from auto-regressive process of price of $p_t = a_0 + a_1 p_{t-1} + v_t$ (v_t:white noise); the new variable is also auto-regressive $\tilde{p}_t = a_1\tilde{p}_{t-1} + v_t$, and fluctuates around zero line.[14]

To investigate stochastic properties of price, calculate auto-covariances of price time series. ML estimate of variance σ_j^2 of stochastic term v_j is calculated easily from regression residual as in $\hat{\sigma}^2 = \sum_{t=1}^{T}\hat{v}_t^2/T$. Next, auto-covariance of 0-th order is calculated $r_0 = \sigma^2/(1-\alpha_1^2)$; auto-covariance of 1-th order $r_1 = a_1 r_0$, and auto-correlation of 1-th order $\rho_1 = (r_1/r_0) = a_1$; finally, auto-covariance of 2-th order $r_2 = a_1 r_1$, and auto-correlation of 2-th order $\rho_2 = r_2/r_0 = a_1^2$. Then, mean distance between peaks (D1) is:

---

14 Solution of non-homogeneous equation has particular solution plus general solution; on seeking particular solution, see, e.g. S.Elaydi(2005). Then, time series of price fluctuates around particular solution on the assumption that disturbance terms of the other price equations are deterministic for a moment so as to split disturbances of the other price equations, unless, price series are on their affects. See, e.g. Chapter 12 of P.J.Dhrymes(1970) for detail. Then, price series obeys univariate normal stationary process.

$$D1 = 2\pi \Big/ \cos^{-1} \frac{(2\rho_1 - 1 - \rho_2)}{2(1-\rho_1)}. \tag{2.15}$$

Mean distance between up-crossing (D2) is:[15]

$$D2 = 2\pi \big/ \cos^{-1} \rho_1 . \tag{2.16}$$

2.5 Determining Wage Rate by Agent-W in a Firm's Internal Labor Market

P.Doeringer and M.Piore(1971) have first advocated internal labor market; turning to the j-th industry, labor demand L_j and wage rate w_j are outcomes of firm's internal labor market of j-th industry, so that monopoly firm decides wage rate.

Wage is cost of living for workers; thus, both minimum level of wage and wage in slide to consumer price change, and thirdly connection to profit maximization are to be considered in wage determination. From the equation (41) of T.Yano and H.Kosaka(2008), capable of interpreting various cases of wage rate determination empirically, we propose again quadratic loss of wage rate presenting the first and the second factors above:

$$-\frac{1}{2} c_j^{w1} (w_j - c_j^{w2} p_c - c_j^{w3})^2 \tag{2.17}$$

c_j^{w3} : minimum level of wage rate p_c : consumer price index

where p_c is defined below:

$$p_c = \sum_{l=1}^{N} \theta_l^{cp} p_l \tag{2.18}$$

$\overline{cp}_l^H$: consumer expenditure of household for l-th product at base year

$$\theta_l^{cp} = \frac{\overline{cp}_l^H}{\overline{cp}^{HT}} \qquad \overline{cp}^{HT} = \sum_{l=1}^{N} \overline{cp}_l^H$$

Therefore, we suppose agent-W has the following individual profit differing slightly from that of agent-P:

$$\max_{w_j} \tilde{\pi}_j^{(w)} = \max_{w_j} \left\{ -\frac{1}{2} c_j^{w1} (w_j - c_j^{w2} p_c - c_j^{w3})^2 + \frac{1}{X_j^*} \left(p_j X_j^D - C_j \right) \right\} \tag{2.19}$$

Given p_j parametrically, i.e. $\partial p_j / \partial w_j = 0$, we have first order condition:

$$\frac{\partial \tilde{\pi}_j^{(w)}}{\partial w_j} = -c_j^{w1} \left(w_j - c_j^{w2} p_c - c_j^{w3} \right) - \frac{1}{X_j^*} \frac{\partial C_j}{\partial w_j} = 0 . \tag{2.20}$$

[15] See H.Kosaka(1976) for detail.

Then we have equation of wage determination by putting $X_j^* = X_j$:

$$w_j = c_j^{w3} + c_j^{w2} p_c - \frac{1}{c_j^{w1}} \frac{1}{\left(X_j / L_j\right)}. \tag{2.21}$$

Wage rate is interpreted as dependent on minimum wage rate, consumer price and labor productivity; consequently, increase of minimum wage rate, consumer price and labor productivity all increase wage rate.

To sum up, producer's behaviors to determine intermediate demands, other factor demands, wage rate and price are endogenized for j-th industry under the principle of profit maximization; as a result, total, marginal, and average costs plus scale economy are also endogenized.

If final demand side models such as consumer expenditure, government expenditure, inventory, and so on for j-th industry are endogenized, we become to know econometric model for j-th industry as a whole; to enlarge industry from one to the whole, we come to possess an econometric general equilibrium model for the national economy. (see Appendix of the total picture of multi-sector system)

Now, to see empirical validity of the above formulated model, we select several industries of the Japanese economy in the next section.

3. Empirical Validity of the Model

3.1 Input Output Data

Institute of Economic Research, Hitotsubashi University of Japan, has been making historical industry data named JIP (Japan Industrial Productivity Database)[16]; we utilizes 2010 version(JIP2010) covering 1970 to 2007 with lack of data 1971-1972; sector classification is one hundred and eight, but 108th sector is unclassified. We select four industries; automobile industry of #54 is facing international high competition; nextly, industries of electricity of #62 and financial institution of #69 are regulated ones; finally, food service industry of #94 in domestic high competition. A highlight here is to employ unified principle of profit maximization in deriving equations of factor demands to price/wage; as a result, total, marginal and average costs and scale economy are calculated endogenously.

3.2 Scale Economy

To begin with, estimate factor demands of one hundred and seven inputs but 108th input plus labor input for each industry[17]; e.g. Table 1 shows estimated results of labor demand equation.

Table 1 Estimated Result of Labor Demand Equation

In the table production is basically statistically significant for all industries; neutral technical progress is also significant except for food service. Employment equation has largely plausible

---

[16] Homepage of Institute of Economic Research, Hitotsubashi Universityof Japan(JIP Database) is ;http://gcoe.ier.hit-u.ac.jp/research/database/index.html#point09

[17] As data of capital stock by sectors are not so reliable, they are not taken in cost function; as a result, total cost are possibly a bit less evaluated.

result. So that, these factor demand functions for multi-sector system would be replaceable by intermediate demands of Leontief fixed coefficient.

Next step is to obtain cost function numerically. Usual way of measuring scale economy may be to investigate elasticity by regression of log-cost on log-production; but, this paper calculates scale economy by definition $SE = AV_j / MC_j$ (MC_j :marginal cost; AV_j :average cost) utilizing estimated generalized Leontief cost function.

Table 2 Scale Economy

On automobile industry of Japanese economy, K.Yoshioka(1989) reported scale value 1.0021~1.0250 based on index theory in use of data 1964-1982; on the other hand, M.A.Fuss and L.Waverman(1992) scale value 1.07 using panel data 1968-1984 for Japan, 1.17 for Canada, 1.09 for US and 1.10 for Germany in their comprehensive study across countries. Table 2 shows computed time series of scale values beginning 1970 to 2007; after the latter half of 80s it shows values more than unity, which is consistent with the other studies.

No empirical studies on scale economy of food service industry are found except ours. Our measurement shows scale figures are all over unity at the outset; paradoxically, smaller scale than those of the other three achieves scale economy with ease by mainly reducing manpower.

On financial institution K.Yoshioka(1989) has also reported, using data of city banks 1974-1984 employing model I among models he proposed; i.e. 1.18946(1974), 1.23539(1975), 1.23900(1976), 1.27315(1977), 1.30416(1978), 1.34333(1979), 1.33003(1980), 1.39042(1981), 1.45630(1982), 1.50585(1983) and 1.51099(1984). Our measurement is shown also in Table 2. After 90s of collapse of bubble economy, new movement of reorganization of financial institution occurred; by removal of ban on bank holding company in 1998 forced banks to go into integration and reorganization; as a result, Mizuho, Yokyo-Mitubishi and Mitsui-Sumitomo (Big Three of Mega Bank in Japan) have emerged; these movement endorses upswinging figures of scale values. On electric utilities J.Nemoto(1992) affirmed scale economy; our analysis also shows scale economy since 1984. However, we have a possibility of estimating distorted cost function in regulated industries in particular.

Then, it might be possible to argue distortion of cost function and factor demands occurred by distorted price as is discussed in S.C.Kumbhkar(1992). We have already incorporate price expectation of current product into its cost function; so that, expected price distorted for high level will probably bring about factor demands distorted for the same direction via "Shishido" effect; if expected price would be converged to marginal cost by perfect competition, proper factor demands would be realized. A gap between distorted and undistorted proper demands is measured in fractional form in the case of employment: $xgap = \exp\left(b_j^{wp}\left(\log p_j^e - \log MC_j\right)\right)/L_j$ (b_j^{wp} : price elasticity of employment). We line four industries from highest gap to lowest; electrical utility, automobile, financial institution and food service; as a result, this fact uncovers food industry has least surplus employees.

3.3 Estimated Results for Price and Wage Rate Equations

Table 3 shows estimated result of price equations.

Table 3 Estimated Result of Price Equations

Upper table of table 3 shows estimated results of (2.11) while lower table reference equation by removing one-period lagged price, i.e. $p_t = d_0 + d_1 MC_t$; for all industries equations (2.11) is superior to the reference equation(see, $R^2(adj.)$ and AIC in the table); consequently, dynamic formulation of price by adding one-period lagged price is justified. It shows coefficients of both marginal cost and one-period lagged price are statistically significant; then, this kind of dynamic price equation is fairly stable.

Table 4 shows result of dynamic stochastic properties of price movement.

Table 4 Dynamic Analysis of Price Movement

Dividing sum of squared residual by number of observation in price equation gives us estimate of disturbance term; next, auto-covariances of 0^{th} up to second order are easily obtained. Now we come to compute the two distances of section 2.4. Relative variability of price is from largest to smallest (see $\sigma/\overline{p}$ in the table); electrical utility 4.0%, financial institution 3.8%, automobile 2.1% and food service 1.2%; as a result, it also leads to relative amplitudes of price time series(see $\sqrt{r_0}/\overline{p}$). Up-crossing interval indicates from the longest to the shortest; food industry 10.7 year, electrical utility 7.36 year, financial institution 6.36 year and automobile 4.38 year while interval between peaks is similar for all industries.

Finally we take wage rate equation in Table 5

Table 5 Estimated Result of Wage Rate Equation

At a glance, for all industries the formulation is appropriate. In particular, labor productivity has statistical significance; this unveils labor productivity in internal labor market is powerful for explaining wage rate, maybe more than unemployment rate of external labor market in interindustry level.

4. Concluding Remarks

Approach we employ here is to set cost function first; secondly, to estimate intermediate demands, labor and capital demands ; thirdly, to estimate indirectly cost function numerically from estimated factor demands; fourthly, to deduce marginal and average costs and scale economy; finally, to estimate wage rate and price determination equations, all under unique principle of profit maximization. The proposed model could be called three agents model where three agents in a firm cooperate each other to determine relevant key variables.

From empirical investigation using data of selected four industries in the Japanese economy, our model is largely consistent with the reality. Enlarging the four industries to the whole economy, and endogenizing final demands such as consumer expenditure and so on may be a next step.

Macroeconomics has been attempting to enter stochastic terms into its economic models under R.Frisch's random shock theory; yet, the model of this paper treats no uncertainty but price; this issue may be another next step.

References

Arndt,C.,S.Robinson and F.Tarp (2002) Parameter Estimation for a Computable General Equilibrium Model: a Maximum Entropy Approach. Economic Modeling,Vol.19,pp.375-398.

Ballard,C.L.,D.Fullerton,J.BShoven and J.Whalley (1985) A General Equilibrium Model for Tax Policy Evaluation. University of Chicago Press.

Chow,G.C. (1973) Problem of Economic Policy from the Viewpoint of Optimal Control, American Economic Review, Vol.63,No.5,pp.825-837.

Christensen,L.R.,D.W.Jorgenson and L.J.Lau(1973) Transcendental Logarithmic Production Function. Review of Economics and Statistics,Vol.55,No.1,pp.28-45.

Deaton,A.S. and J.Muellbauer (1980) An Almost Ideal Demand System. American Economic Review,Vol.70,312-326.

Dhrymes,P.J. (1970) Econometrics:Statistical Foundations and Applications,Harper & Row,Publishers:New York.

Dixon,P.B, B.R.Parmenter, A.A.Powell and P.J.Wilcoxen (1992) Notes and Problems in Applied General Equilibrium Economics. North-Holland.

Diewert,W.E. and K.J.Fox (2004) On the Estimation of Returns to Scale, Technical Progress and Monopolistic Markup. Discussion Paper No.04-09,Department of Economics, University of British Columbia.

Diewert,W.E. and K.J.Fox (2008) On the Estimation of Returns to Scale, Technical Progress and Monopolistic Markups. Journal of Econometrics, 145,pp174-193.

Doeringer,P. and M.Piore (1971) Internal Labor Markets and Manpower Analysis. Lexington, Mass.D.C. Heath and Company.

Duesenberry,J.S, G.Fromm, L.R.Klein and E.Kuh(eds.) (1965) The Brookings Quarterly Eco nometric Model of the United States. Chicago:Rand-McNally and Company.

Elaydi,S.(2005) An Introduction to Difference Equations. Springer.

Fuss,M. (1977) The Structure of Technology Over Time. Econometrica,Vol.45,pp.1797-1822.

Fuss,M.A. and L.Waverman (1992) Costs and Productivity in Automobile Production. Cambridge University Press.

Hall,R.E. (1988) The Relation between Price and Marginal Cost in the U.S. Industry. Journal of Political Economy, Vol.96,No.5,pp.921-947.

Harris,R. (1984) A General Equilibrium Analysis of Small Open Econoomies with Scale Economies and Imperfect Competition. The American Economic Review,Vol.74,No.5,pp.1016-1032.

Johansen,L. (1960) A Multi-Sector Study of Economic Growth. North-Holland, Amsterdam.

Jorgenson, D.W. (1984) Econometric Methods for Applied General Equilibrium Analysis. In Scarf,H.E. and J.B.Shoven (eds.) Applied General Equilibrium Analysis. New York,Cambridge University Press.

Jorgenson,D.W. and K.Y.Yun (1990) Tax Reform and U.S. Economic Growth. Journal of Political Economy,Vol.98,No.5,Part 2:The Problem of Development: A Conference of the Institute for the Study of Free Enterprise System(Oct.,1990),pp.S151-S193.
Jorgenson, D.W. and D.T. Slesnick (1997) General Equilibrium Analysis of Economic Policy. in Welfare. D.W.Jorgenson(ed.) Volume 2. Cambridge,The MIT Press.
Jorgenson,D.W., H.Jin, D.T.Slesnick and P.J.Wilcoxen (2012) Chapter 17: An Econometric Approach to General Equilibrium Modeling, In P.B.Dixon and Jorgenson,D.W.(eds.) (2012) Handbook of Computable General Equilibrium Modeling. North-Holland.
Kolstad,C.D. and JT.Lee (1993) The Specification of Dynamics in Cost Function and Factor Demand Estimation. Review of Economics and Statistics,Vol.75,No.4,721-726.
Kosaka,H. (1976) Multivariate Normal Distributions and their Applications to the Dynamic Properties of Macro-Econometric Models. Keio Economic Studies,Vol.13,No.1,pp69-78.
Kosaka,H. (2011) Multi-Country and Multi-Sector Modeling for the World Economy. G-SEC Working Paper,No.29,Keio University.
Kratena,K. (2005) Price and Factor Demand in an Endogenized Input-Output Model. Economic System Research,Vol.17,No.1,pp.47-56.
Kulatilaka,N. (1987) The Specification of Partial Static Equilibrium Model. Review of Economics and Statistics,Vol.69,No.2,327-335.
Kumbhkar,S.C. (1992) Allocative Distortions, Technical Progress, and Input Demand in U.S. Airlines:1970-1984. International Economic Review, Vol.33,No.3,pp.723-737.
Leontief,W. (1951) The Structure of the American Economy. Oxford University Press:New York.
H.Lofgren,R.L.Harris and S.Robinson (2002) A Standard Computable General Equilibrium(CGE) Model in GAMS. International Food Policy Research Institute.
McKitrick, R.R. (1998) The Econometric Critique of Computable General Equilibrium Modeling: The Role of Functional Forms. Economic Modelling,Vol.15,pp.543-573.
Mongelli,I., F.Neuwahl and J.M.Rueda-Cantuche (2010) Integrating a Household Demand System in the Input-Output Framework. Economic system Research,Vol.22,No.3,pp.201-222.
Morrison,C. (1988) Quasi-fixed Inputs in the U.S. and Japanese Manufacturing: a Generalized Leontief Restricted Cost Function Approach. Review of Economics and Statistics,Vol.70,No.2,275-287.
Nakamura,S. (1990) A Nonhomothetic Generalized Leontief Cost Function Based on Pooled Data. The Review of Economics and Statistics Vol.72, No.4, pp.649-656
Negishi,N. (1961) Monopolistic Competition and General Equilibrium. Review of Economic Studies,Vol.28,No.3,pp.196-201.
Nemoto,J. (1992) Economies of Scale in Electric Utilities: A Survey of Recent Studies. Journal of Socio Economic Research,No.31,pp.15-24(in Japanese).
Phillips,A.W. (1954) Stabilization Policy in a Closed Economy. Economic Journal,Vol.64,No.254,pp.290-323.
Pindyck,R.S and J.J.Rotemberg (1983) Dynamic Factor Demands and the Effects of Energy Price Shocks. American Economic Review,Vol.73,No.5,1066-1079.
Shibata,T. and H.Kosaka (2011) Modeling for the Nine Interregional System of the Japanese Economy,SFC-RM2011-003.
Shoven,J.B. and J.Whalley (1972) A General Equilibrium Calculation of the Effects of

Differential Taxation of Income of from Capital in the US. Journal of Public Economics,Vo.1,281-322.
Shoven,J.B. and J.Whalley (1973) General Equilibrium with Taxes:A Computational Procedure and an Existence Proof. Review of Economic Studies,Vo.40,475-490.
Shoven,J.B. and J.Whalley (1992) Applying General Equilibrium. Cambridge University Press.
Saito,M. (1971) An Interindustry Study of Price Formation. Review of Economics and Statistics,Vol.53,pp.11-25.
Shibata,T. and H.Kosaka (2011) Modeling for the Nine Interregional System of the Japanese Economy. SFC-RM2011-003.
Shishido,S. and O.Nakamura (1992) Induced Technical Progress and Structural Adjustment: A Multi-Sectoral Model Approach to Japan's Growth Alternatives. Journal of Applied Input Output Analysis,Vol.1,No.1.
Tsujimura,K. and M.Kuroda (1974) General Equilibrium Analysis for the Japanese Economy. Chikuma:Tokyo(in Japanese).
Tsutsui,Y. (2005) Competition and Efficiency in Banking System. Toyo Keizai:Tokyo(in Japanese)
Turnovsky,S.J (1977) Macroeconomic Analysis and Stabilization Policy. Cambridge University Press, Cambridge.
Wharton Econometric Forecasting Associates (1982) The Wharton Long-term Model-Structur e and Specification-. Unpublished Monograph.
Yano,Y. and H.Kosaka (2008) National-Currency Based International Input-Output Analysis: Data Construction and Model Structure. G-SEC Working Paper No.24.
Yoshioka,K. (1989) Productivity Analysis in Japanese Manufacturing and Financial Institution Industries. Toyo Keizai:Tokyo(in Japanese).

Appendix: Total Picture of Multi-Sector System

In this appendix, we intend to elucidate the total picture of our multi-sector system.

a) Demand Side

The following is well known equilibrium of market of goods and service:

$$X_i = \sum_{j=1}^{N} x_j^i + cp_i^H + cp_i^N + cg_i + ifp_i + ifg_i + inv_i + ex_i - im_i \qquad (A.1)$$

x_j^i : intermediate demand by j-th industry for i-th product

cp_i^H : consumer expenditure of household for i-th product

cp_i^N : consumer expenditure of non-household for i-th product

cg_i : government expenditure for i-th product

inv_i : inventory investment for j-th product

ifp_i: private fixed investment for i-th product

ifg_i: government fixed investment for i-th product

ex_i: export of i-th product im_i: import of i-th product

where intermediate demands are already explained:

$$x_j^i = b_j^{ii}\left(X_j\right)^{b_j^{Xi}} \exp\left(b_j^{ti} t\right)\left(p_j\right)^{b_j^{pi}}. \quad \text{(A.2)}$$

Next, household designs expenditure allocating wealth W over expenditure by goods where (N+1)th goods are for future expenditure(i.e. current saving) under maximizing CES utility:

$$\max_{cp_i^H\,(i=1,..N)} U = \max_{cp_i^H\,(i=1,..,N)} \left(\sum_{i=1}^{N+1} \alpha_i \left(cp_i^H\right)^{-\rho}\right)^{-1/\rho} \quad \text{(A.3)}$$

subject to $W = \sum_{j=1}^{N+1} w_j L_j + S_{-1} = \sum_{i=1}^{N+1} p_i cp_i^H$ S_{-1} :saving at the beginning

where $\sum_{i=1}^{N+1} p_i cp_i^H = \sum_{i=1}^{N} p_i cp_i^H + p_c cp_{N+1}^H = \sum_{i=1}^{N} p_i cp_i^H + S$ holds.

Then optimization problem (A.3) yields optimal expenditure:

$$cp_i^H = \frac{W\left(\dfrac{\alpha_i}{p_i}\right)^{\sigma}}{\sum_{l=1}^{N} \alpha_l^{\sigma} p_l^{1-\sigma}} \qquad \sigma = \frac{1}{1+\rho} : \text{elasiticity of substitution} \quad \text{(A.4)}$$

From (A.4) we have following equation with a bit complication in estimation:

$$\log\left(\frac{cp_i^H}{W}\right) = \sigma \log\left(\frac{\alpha_i}{p_i}\right) - \log\left(\sum_{l=1}^{N} \alpha_l^{\sigma} p_l^{1-\sigma}\right) \quad \text{(A.5)}$$

For non-household expenditure cp_i^N , similar argument may be possible; however, expeditures of the others are left unexplained in this article.

b) Supply Side

From (2.4) we obtain labor and capital demands:

$$L_j = b_j^L\left(X_j\right)^{b_j^{LX}} \exp\left(b_j^{Lt} t\right)\left(p_j\right)^{b_j^{Lp}} \qquad j = 1,2,..,N \quad \text{(A.6)}$$

$$K_j = b_j^K\left(X_j\right)^{b_j^{KX}} \exp\left(b_j^{Kt} t\right)\left(p_j\right)^{b_j^{Kp}} \qquad j = 1,2,..,N \quad \text{(A.7)}$$

Total capital demand $K_t = \sum K_j(t)$ might be connected to ifp_j by definitional relation.

Equations (A.2) together with (A.6) and (A.7) make cost function, which derives marginal cost; then, the marginal cost is non-linear function of production, all the prices and all the wage rates plus capital cost/time trend.

$$MC_t = MC\left(X_t, p_t, w_t, r, t\right) \quad \text{(A.8)}$$

$$MC_t = \left(MC_1(t) \quad MC_2(t) \quad . \quad MC_N(t)\right)' \text{: column vector of marginal cost}$$

$$p_t = \left(p_1(t) \quad p_2(t) \quad . \quad p_N(t)\right)' \text{: column vector of price}$$

$$w_t = \left(w_1(t) \quad w_2(t) \quad . \quad w_N(t)\right)' \text{: column vector of wage rate}$$

Equation (2.11)(j=1,2,..,N) constitute dynamic simultaneous equations of prices:

$$p_t = A_0 + A_1 p_{t-1} + A_2 MC_t \qquad \text{(A.9)}$$

$$A_0 = diag\left\{\mu_j + \left(1-\beta_j\right)/c_j^{p1}\right\} \qquad A_1 = diag\left\{c_j^{p2}\right\} \qquad A_2 = diag\left\{\beta_j \delta_j / c_j^{p1}\right\}$$

In the final, we deduce wage rate equation from (2.21):

$$w_t = B_0 + B_1 p_c(t) - L_t' B_2 XI_t \qquad \text{(A.10)}$$

$$L_t = \left(L_1(t) \quad L_2(t) \quad . \quad L_N(t)\right)' \text{: column vector of labor demand}$$

$$XI_t = \left(1/X_1(t) \quad 1/X_2(t) \quad . \quad 1/X_N(t)\right)' \text{: column vector}$$

$$B_0 = diag\left\{c_j^{w3}\right\} \qquad B_1 = diag\left\{c_j^{w2}\right\} \qquad B_2 = diag\left\{1/c_j^{w1}\right\}$$

c) other variables

By definition whole price p^T leads to:

$$p^T(t) = \sum_{i=1}^{N}\left(\frac{\overline{X}_i}{\overline{X}^T}\right)p_i(t) = \sum_{i=1}^{N}\theta_i^X p_i(t) \qquad \overline{X}^T = \sum_{i=1}^{N}\overline{X}_i \qquad \text{(A.11)}$$

$\overline{X}_i$: production of i-th product at base year

Also unemployment is elucidated by definition at macro-level:

$$UN_t = L_t^S - \sum_{j=1}^{N} L_j(t) \qquad L_t^S \text{: labor supply(exogenous)} \qquad \text{(A.12)}$$

Table 1 Estimated Result of Labor Demand Equation

$$\log(L_j) = b_j^w + b_j^{wX}\log(X_j) + b_j^{wt}t + b_j^{wp}\log(p_j^e)$$

	Automobile	Electric Utility	Financial Institution	Food Service
Coefficient b_j^w	8.634438 (3.361388)	1.147394 (1.119812)	8.311416 (15.07948)	-1.579562 (-0.840391)
Coefficient b_j^{wX}	0.218228 (1.412035)	0.659101 (10.46663)	0.334101 (10.13691)	0.983107 (8.865727)
Coefficient b_j^{wt}	-0.009579 (-1.89620)	-0.029020 (-13.09512)	-0.011381 (-6.239821)	
Coefficient b_j^{wp}	0.269584 (1.491285)	0.018381 (0.938936)	0.469213 (11.88388)	0.003562 (0.043583)
$R^2(adj.)$	0.763903	0.895417	0.939518	0.975309
AIC	-3.784647	-4.167197	-4.105404	-3.734408

Note: Figures in parentheses are t-values; price expectation $p_j^e = p_j$ is assumed, and disturbance term of equation of automobile industry is assumed to obey first order auto-regressive process. Time trend is removed for food service industry on account of positive sign.

Table 2 Scale Economy

	Automobile	Electric Utility	Financial Institution	Food Service
1970	0.744	0.858	0.564	1.035
1973	0.825	0.947	0.642	1.095
1974	0.841	0.921	0.630	1.125
1975	0.853	0.904	0.659	1.176
1976	0.883	0.919	0.677	1.182
1977	0.907	0.953	0.707	1.200
1978	0.948	0.991	0.739	1.268
1979	0.947	1.018	0.750	1.295
1980	0.960	0.950	0.756	1.269
1981	0.970	0.961	0.754	1.246
1982	0.958	0.961	0.765	1.250
1983	0.966	0.990	0.802	1.257
1984	0.975	1.024	0.844	1.262
1985	0.996	1.052	0.875	1.291
1986	1.005	1.099	0.929	1.299
1987	1.008	1.140	1.013	1.310
1988	1.018	1.179	1.046	1.330
1989	1.038	1.193	1.072	1.310
1990	1.049	1.215	1.052	1.307
1991	1.041	1.257	1.025	1.313
1992	1.038	1.292	1.059	1.327
1993	1.029	1.318	1.078	1.343
1994	1.016	1.367	1.098	1.373
1995	1.014	1.386	1.131	1.381
1996	1.012	1.391	1.106	1.399
1997	1.021	1.379	1.124	1.378
1998	1.003	1.398	1.102	1.360
1999	1.005	1.402	1.124	1.353
2000	1.017	1.380	1.132	1.319
2001	1.016	1.373	1.160	1.339

2002	1.021	1.363	1.174	1.340
2003	1.018	1.353	1.166	1.329
2004	1.022	1.338	1.153	1.329
2005	1.026	1.315	1.164	1.341
2006	1.035	1.289	1.169	1.356
2007	1.037	1.294	1.162	1.371

Table 3 Estimated Result of Price Equation

$$p_t = a_0 + a_1 p_{t-1} + a_2 MC_t \ : \ a_0 = \mu_j + (1-\beta_j)/c_j^{p2} \ ; a_1 = c_j^{p2} \ ; a_2 = \beta_j \delta_j / c_j^{p1}$$

	Automobile	Electric Utility	Financial Institution	Food Service
Coefficient a_0	0.596104 (7.113058)	0.066845 (1.727630)	0.349117 (3.990644)	0.075921 (5.565345)
Coefficient a_1	0.136843 (1.558748)	0.658036 (17.93582)	0.550612 (6.746083)	0.833089 (21.96160)
Coefficient a_2	0.374255 (7.359178)	0.697241 (8.621290)	0.345868 (4.365047)	0.180258 (2.013924)
$R^2(adj.)$	0.718165	0.953339	0.695643	0.995733
AIC	-4.53002	-3.410484	-3.12119	-6.137198

Note: MC_t =marginal cost of industry

Reference equation $p_t = d_0 + d_1 MC_t$

Coefficient d_0	0.605316 (8.50233)	0.291694 (2.605570)	1.002378 (12.51344)	-0.108800 (-3.142110)
Coefficient d_1	0.518105 (6.595632)	1.700720 (6.599043)	0.121157 (0.636869)	2.105965 (27.52660)
$R^2(adj.)$	0.548401	0.548663	-0.017276	0.955792
AIC	-3.369337	-0.733786	-1.244797	-3.419860

Note: AIC is directly incomparable between the two because of difference of number of observation; but, it is possible to compare them roughly.

Table 4 Dynamic Analysis of Price Movement

Industry	Automobile	Electric Utility	Financial Institution	Food Service
Covariance σ^2	0.000529	0.0016208	0.001576	0.000106
$\sigma/\bar{p}$	0.021478	0.039927	0.037751	0.012493
Normality Test(Jarque-Bera)	19.90899	36.81236	6.408623	1.051244
Autocovariance of 0th order r_0	0.000539	0.002859	0.002262	0.000346
$\sqrt{r_0}/\bar{p}$	0.021680	0.053028	0.045227	0.022572
Autocorrelation of 1st order ρ_1	0.136843	0.658036	0.550612	0.833089
Autocorrelation of 2nd order ρ_2	0.018726	0.426521	0.303174	0.694037
Distance between Peaks	3.114963	3.625507	3.495555	3.797870
Distance between Up-Crossings	4.382910	7.369320	6.361249	10.71951

Note: $\bar{p}$=sample average of price

Table 5　Estimated Result of Wage Rate Equation

$$\log(w_j) = c_j^{w3} + c_j^{w2} p_c - \frac{1}{c_j^{w1}} \left(\frac{1}{X_j / L_j} \right)$$

	Automobile	Electric Utility	Financial Institution	Food Service
Coefficient c_j^{w3}	10.84166 (16.84955)	13.72033 (43.81231)	1.068802 (1.825868)	1.630379 (3.056200)
Coefficient c_j^{w2}			6.574029 (12.88584)	1.751240 (13.63969)
Coefficient $1/c_j^{w1}$	211.0725 (7.672511)	457.7559 (22.08166)	21.25327 (10.66715)	11.66447 (3.514986)
$R^2(adj.)$	0.623119	0.932899	0.983848	0.852012
AIC	4.112569	2.345086	-0.150641	-1.554045

Note: Consumer price is omitted for automobile and electric utility industries because of negative sign.